OPTIMAL CONFIGURATION OF A DISTRICT HEATING NETWORK THROUGH THERMOECONOMICS

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ABSTRACT

An issue in planning district heating systems is the network extension. The network does not connect necessarily all the buildings in the town. The selection of the areas to be connected to the network should be made during the design process, since further variations in the planned topology are usually difficult and conduct to a non-optimal configuration.

This paper deals with the synthesis of the district heating network using economic criteria, so that the optimal economic configuration is determined. The algorithm here proposed is based on the application of thermoeconomics, which allows one to reduce significantly the time required for solving the problem.

An initial superstructure connecting all the possible users is built. Then the structure is progressively simplified, until the optimal configuration is found. The algorithm for the network simplification is based on a simulated annealing technique. The procedure is applied to the design of a district heating network in a small portion of the urban area of Turin. The choice is made in order to built a significant case study in order to test the procedure. The data for the network simulation are obtained from the real system.

INTRODUCTION

District heating is a rational way to provide heat to multiple users located in a urban area. The system is constituted by a single or a few centralized plants, generally CHP and high efficiency boilers, which fed the network. The main expected advantages are reduced emissions of air pollutants as well as reduced resource depletion, with respect to distributed thermal generation.

The main issue on this kind of system is referred to its cost. As any other system it needs to be more convenient than the alternatives. A second issue must be considered: building these systems requires several years and cause lots of problems to the community. For this reason, the system must be designed in its final structure, with few possibilities of making changes. In particular, design involves the choice of the possible users to be connected, the topology and the pipe diameter of each branch.

Such a problem can be solved as a synthesis problem, i.e. an optimization where the system structure is not defined a priori [1]. In this way it is possible to define the optimal network that minimizes (or maximizes) an objective function, such as the minimum cost of heat or the maximum benefit.

This paper aims at proposing a thermoeconomic approach for determining the optimal network extension (the number and which users are connected) in terms of general cost, monetary, environmental, etc. The main advantage of such procedure is that it does not need big computational efforts for the network synthesis.

The theoretical considerations are applied to a network which does not exist in the reality, nevertheless the possible users are constituted by the buildings of an area in the west part of Turin, close to the area at the moment really connected with the DHN. In additions, all the data corresponding to the thermal load and geometric characteristics of the network have been derived from the Turin district heating network.

The thermal plant is considered in the centre of this area. It is constituted by a cogenerative combined cycle and some boilers. The CHP is designed in order to provide 40% of the maximum thermal load, which corresponds to more than 70% of the annual thermal energy request. Cogeneration is obtained through a steam extraction at about 2.5 bar from the turbine, which feeds a heat exchanger. The remaining request is covered by means of boilers [2]. These percent values are assumed to be independent on the number of users and, thus, on the total load. Such assumption is possible when the thermal plant design is made after the determination of the optimal network extension. Using this information, it is possible to estimate the unit cost of heat provided to the district heating network, which is an essential information for the economic analysis.

For the network synthesis, some simplifications are made. The users are grouped together, in reason of geographic criteria, into zones. Each zone is represented on the network map as a point called thermal barycentre. A tree-shaped sub-network connects each single user of the area to the corresponding thermal barycentre. The DHN is a closed network; water temperature in the outgoing pipes is assumed at about 120 °C, while in the return pipes is assumed at about 60 °C, being load variations mainly controlled by operating on the water mass flow rate. A heat exchanger located at each single user operates the connection between the main network and the building distribution system. Water circulation through the network is obtained by means of several pumps, generally located in correspondence of the main ramifications.

The connection of users to the network is decided in reason of economic criteria: if the unit cost of heat provided to the user through the DHN is higher than possible alternatives (local boilers or micro cogeneration systems) the user is not connected to the network. For the users not connected to the network a local boiler provides heat to the users.

This calculation is made complex by the fact that the elimination of a user affects the unit cost of heat provided to the other users.

SYSTEM MODEL

The optimal synthesis of energy systems is approached by starting with a superstructure [1], which is a DHN involving all of the possible zones and thus all the users. The technique is widely used for solving synthesis problems (see for example [3-5]). Once the superstructure is built, the problem can be solved as an optimization problem. When the optimal mass flow rate in a pipe is zero, it means that the pipe must be eliminated from the structure.

The procedure begins with the evaluation of the objective function in the initial structure, corresponding to all the users connected with the network. The network is then reduced. The elimination method, based on probabilistic criteria, will be explained further on. The aim of the elimination is to disconnect the users that determine high costs and the corresponding pipes connecting these users with the rest of the network. For the disconnected users heating is obtained with the most convenient alternative (in this paper the possible alternatives have not been investigated further and heat is provided by means of local boilers). The procedure is stopped when all the users are eliminated; it is not safe to stop the procedure when a minimum/maximum is reached because of the possible presence of local minima/maxima.

The details of the selected procedure are shown by considering as the objective function to be minimized the average unit cost of heat provided to the users.

The first step consists then in calculating the unit cost of heat as:

$$\bar{c} = \frac{C_{tot}}{Q_u} = \frac{C_{net} + c_F \cdot Q_F + c_P \cdot L_p}{Q_u} \tag{1}$$

The cost of network C_{net} includes purchase of insulated pipes, heat exchangers, pumps, valves, together with other direct costs, like excavation, installation and paving restoration. Indirect costs, such as engineering, legal costs, contingency, insurance, as well as additional costs are also included; these costs have been estimated on the basis of the purchasing costs [6]. Maintenance is also included.

C_{net} is an annual cost. Year is the best unit time to be used for the techno-economic (or thermoeconomic) analysis of such system, since the production varies depending on the external temperature. Annual cost is calculated as an ordinary annuity of the total investment cost, which depends on the total life of the plant and the rate of return.

The unit cost of heat exchanged between thermal plant and DHN has been calculated starting from the electricity cost of a non-cogenerative combined cycle and then assigning the cost of non produced electricity to the heat. This occurs because steam extraction reduces the electricity production. Moreover, the unit cost of electricity has been considered as linearly dependent on the plant size.

This procedure for the calculation of the unit cost of heat is not general; a thermoeconomic analysis of the power plant could provide more rational results.

Heat request by the users Q_u and heat supplied by the thermal plant Q_F differ because of heat losses. This difference has been evaluated as 6% of the heat request during a year. The losses have been internalized in the unit cost of heat supplied to the network so that Q_u and Q_F can be assumed coincident.

The last term at numerator of equation (1) accounts for the electricity cost for pumping, being c_P the unit cost of electricity and L_p the annual electricity consumption, calculated as:

$$L_p = \frac{1}{\eta_p} \int_{year} G \cdot \frac{\Delta p}{\rho} \cdot dt \tag{2}$$

where η_p is the average pump efficiency, G is the water mass flow rate, ρ is the water density (assumed constant) and Δp the total pressure losses due to pipe friction and localized resistances.

The terms in equation (1) depend on the thermal load supplied by the network and on its extension. The first step of the analysis consists in selecting the localization of the thermal plant (TP). This is generally subjected to multiple constraints, technical, social, etc. The possible urban area heated by the thermal plant must be chosen as well. This area can be divided into zones, each including one or more buildings. The number of zones should be selected as trade-off between result accuracy (large number of zones) and time required for design and calculation (small number of zones). For each zone, the total volume of buildings is determined. The thermal barycentre can be easily located in the area by considering the position of buildings and their respective volume (the geometric barycentre can be used as well, especially when the building structure in the zone is sufficiently regular). At this point, the network connecting the thermal plant with thermal barycentres can be traced. In the case here analyzed, the initial superstructure is constituted of a total volume of buildings equal to 25×10^6 m^3. This area has been divided into 72 zones, connected with the thermal plant as shown in figure 1.

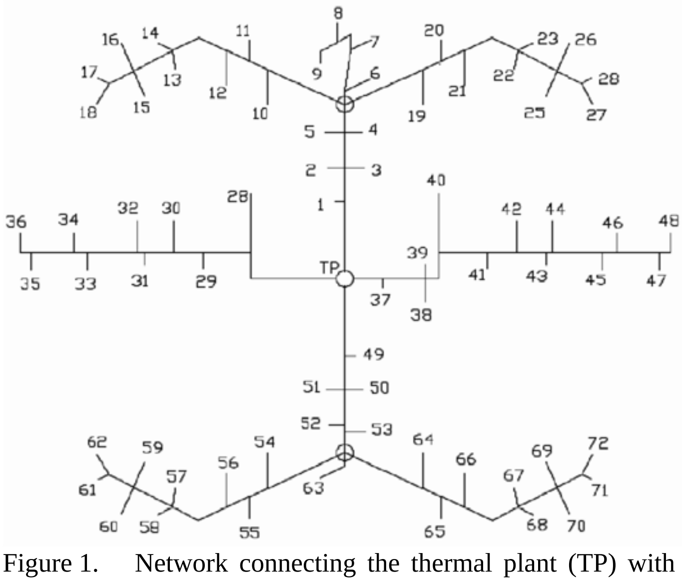

Figure 1. Network connecting the thermal plant (TP) with all the possible zones in the area.

The annual heat load of each single zone Q_z is calculated by considering, for the whole heating season, the daily difference between the internal temperature (20 °C) and the external temperature, the average thermal transmittance of buildings (through walls, windows, floor, etc.) and the number of daily heating hours (hh). The thermal transmittance of building can be multiplied for a shape factor defined as the ratio of external surface and building volume; this quantity, here indicated as r, expresses the volumetric heat losses per unit temperature difference. Its value has been calculated experimentally for several buildings; an average value of 0.9 W/(m^3K) has been assumed. This value has been calculated through experimental analysis of several buildings over a year.

The annual heat load for a zone in kWh is then expressed as

$$Q_z = \frac{r \cdot DTD \cdot hh \cdot V_z}{1000} \quad (3)$$

where V_z is the total volume of buildings in the zone and DTD is the summation of daily difference between internal and external temperature, calculated for the whole heating season. This value is available for all the Italian municipalities but can be easily calculated from historical data. The number of daily heating hours is established by law, depending on DTD. For the town of Turin, DTD is 2014 °C·day, being the heating season from the middle of October to the middle of April, while the number of heating hours is 12 per day.

The total heat load is calculated as summation of the contributions of all the zones. The network operates longer than specified, in particular because of the non-contemporary request by the different users. In this analysis, the total load calculated by means of equation (3) has been considered as spread on 18 hours per day during the heating season. In addition, the thermal flow outside this period has been assumed 11% of the maximum thermal flow. This last assumption has been formulated by making use of experimental data.

The purchase cost of the DHN is calculated by considering the contributions of the insulated pipes constituting the main network (from the thermal plant to each thermal barycentre), the insulated pipes of each sub-network (from the barycentre to the buildings on the zone), the pumps, the special components, such as valves and junctions between pipes, the heat exchangers in the buildings and in the thermal plant.

The purchase cost of the insulated pipes is expressed through a polynomial function, obtained by interpolating available data:

$$PC_{IP} = \left(a_0 + a_1 \cdot D_{int} + a_2 \cdot D_{int}^{\ 2}\right) \cdot 1.25 \cdot L \cdot 2 \quad (4)$$

where D_{int} is the internal diameter and L its length, 1.25 is a corrective factor used to include the cost of special components also determined trough available data and the coefficient 2 accounts for the double pipe. The calculated values of polynomial coefficients are: a_0=6.86 €, a_1=0.31 €/mm, $0.4 \cdot 10^{-3}$ €/mm^2.

The internal diameter is calculated by first determining the mass flow rate in each branch. The mass flow rate is imposed by the thermal requirement of each user downstream that branch:

$$\Phi = G \cdot \left(h_o - h_r\right) \quad (5)$$

where Φ is the thermal flow provided to the users (the maximum load is considered in design), G the water mass flow rate, h_0 and h_r the enthalpies of fluid feeding and returning from the users. The diameter is determined by imposing the maximum velocity v_{max} allowed in the pipes. This value is mainly defined on the basis of an economic criterion, since friction losses and, consequently, pumping cost depend on the square of velocity. On the other hand, a too low velocity would determine a large pipe diameter, thus high investment costs. In this analysis a value of 2.5 m/s is considered. The water mass flow rate G is expressed as:

$$G = \rho \frac{\pi D_{int}^{\ 2}}{4} v_{max} \quad (6)$$

The sub-network length is calculated by using an empirical formulation obtained by interpolating five hundred cases randomly generated by varying the number of buildings on a rectangular shaped zone and its area [7].

The purchase cost of the heat exchangers has been calculated as function of the heat transfer area, according with a general function [6]:

$$PC_i = PC_0 \cdot \left(\frac{X_i}{X_0}\right)^{\alpha} \quad (7)$$

where PC_0 is the known cost of the device at a specific size, X is a variable selected for expressing the component size, X_i is its value for the device whose cost is calculated and X_0 its reference value. For heat exchangers the variable expressing the component size is the heat transfer area. Reference values PC_0 and X_0 are respectively assumed 187 € and 0.65 m^2, while is α=0.66.

A similar expression is considered for the pumps. As variable for the component size the electric power is considered. Reference values PC_0 and X_0 are respectively assumed 35000 € and 135 kW, while α=0.65.

The costs for the installation have been calculated by determining the dimensions of the excavation. It has to be 500 mm wider than the pipes' external diameter and 650 mm deeper; a sustaining and covering layers of sand 100 mm high is also required. The specific costs of sand, excavation work and paving restoration (material and work) have been assumed respectively 18 €/m^3, 5.16 €/m^3 and 10.33 €/m^2. An example of excavation is shown in figure 2.

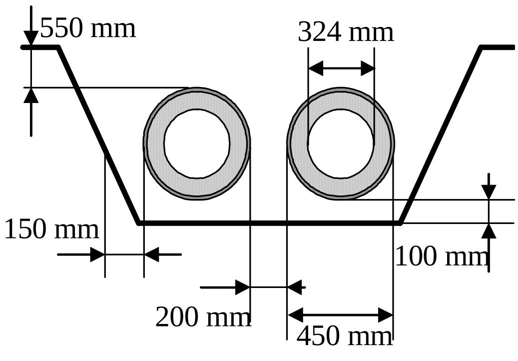

Figure 2. Schematic of the excavation.

The costs for the control system and the insurance during the network's construction and the engineering, legal and contingency costs are calculated as percentages of the direct costs. The total percentage of these costs is 20%. Maintenance and insurance costs for the DHN can be assumed as 3.2% of the capital costs, while the maintenance cost of the heat exchangers has been considered 5% of the purchase cost.

Both capital and operational costs have been amortized. For the first ones a discount rate of 6% has been considered. The equivalent annual cost has been computed as

$$C_C = \frac{(1+d)^l}{(1+d)^l - 1} \cdot d \quad (8)$$

in which d is the discount rate and l is the life of the network in years.

When considering the operational costs, that are paid each year, an inflation rate has to be involved in the economic analysis. In this paper the value of the inflation rate is assumed 2.5% and the operational costs are calculated as

$$C_{OP} = P_0 \cdot \frac{k \cdot (1 - k^l)}{1-k} \cdot \frac{d \cdot (1+d)^l}{(1+i)^l - 1} \quad (9)$$

$$k = \frac{1+i}{1+d} \quad (10)$$

P_0 is the annual operational cost estimated at the beginning of the first year. Equation (9) allows one calculating a constant annuity all along the life of the system, taking into account that the cost P increases each year due to inflation.

The overall equivalent annual cost of the asset is given by the summation of equivalent annuity for capital and operational expenses, namely:

$$C_{net} = C_C + C_{OP} \quad (11)$$

The average unit cost of heat has been then calculated by using equation (1).

THERMOECONOMIC SYNTHESIS

A thermoeconomic analysis is implemented for the designed network, where all the possible users are connected. In particular, a useful approach that can be adopted for this purpose is that proposed by Valero and co-workers in the eighties [8, 9]. One of its main characteristics is the matrix based calculation: the incidence matrix is used for expressing the equations of cost conservation for all the components (pipes, heat exchangers, pumps). The incidence matrix (see for example [10]) was formulated in the ambit of the graph theory [11], which is widely adopted for the topology definition as well as the fluid dynamic and thermal calculation of distribution networks [12]. The incidence matrix, **A**, is characterised by as many rows as the branches (*m*) and as many columns as the nodes (*n*). The general element A_{ij} is equal to 1 or –1, respectively if the branch *j* is entering or exiting the node *i* and 0 in the other cases. The use of the incidence matrix allows one to express the balance equation of the flow of the general extensive quantity $\mathbf{G_x}$ as:

$$\mathbf{A} \cdot \mathbf{G}_x + \mathbf{G}_{x_d} = \mathbf{0} \quad (12)$$

where $\mathbf{G}_x$ is the vector containing the values assumed by the quantity G_x in the nodes and $\mathbf{G}_{xd}$ is the vector that allows to account for the amount destructed in the branches, if non null. In thermoeconomics, equation (8) allows one writing the cost balance:

$$\mathbf{A} \cdot \Pi + \mathbf{Z} = \mathbf{0} \quad (13)$$

Π is the vector containing the cost of all the flows, while Z contains the cost rate of the components. The calculation of all the costs requires the formulation of n-m auxiliary equations, which are obtained through definition of resources and products of each component, expressed in terms of exergy flows [13]. The auxiliary equations were formulated as four propositions. In the case of application to district heating networks, only two of them are required [14]: 1) the unit cost of an exergy flow entering the system from the environment is assumed equal to its unit price; 2) the unit cost of all the flows exiting a bifurcation is the same.

The unit cost of a flow *c* can then be calculated, by dividing the costs for the corresponding exergy flow:

$$c = \frac{\Pi}{\Psi} \quad (14)$$

where Π is the thermoeconomic cost of the flow and Ψ its exergy.

At this point, the unit cost for each user, can be calculated. This cost is not the same for all of them because of the different exergy destruction (mainly due to friction) and the pipe cost associated to the different paths joining the thermal plant with the users.

The synthesis procedure adopted in a former paper [7] was iterative. Each iteration consisted of four steps: 1) calculation of the unit cost of all the users connected to the actual network configuration; 2) elimination of the user characterized by the highest cost and the corresponding piping joining the user with the rest of network; 3) calculation of the new structure obtained without considering the eliminated zone. The calculation consists in determining the mass flow rates in the pipes, the corresponding diameters and the total heat load (equation 3). All the costs are recalculated as well; 4) calculation of the objective function for the new configuration, i.e. the average cost of heat provided to the users. Then, the procedure restarts from step 1).

The main advantage of such procedure is that large structures characterized by hundreds of users can be easily processed; the computational time is linearly dependent on the number of decision variables, i.e. the number of users. In contrast, a disadvantage is that it does not guarantee the obtainment of the true optimum. This event can occur when several users in different zones of the network are characterized by values of the unit costs close to that of the user which is eliminated at the end of the current iteration.

SIMULATED ANNEALING

The procedure formerly proposed allows one to obtain a quasi-optimal structure, in general just slightly different from the true optimal structure. In this paper, a new iterative method is proposed and applied. The main difference with respect to the previous approach is the use of probabilistic criteria for simplifying the network while the iterations proceed. The idea at the basis of this method adopted is known as Simulated Annealing [15]. The method adopted here is slightly different from the true Simulated Annealing.

Annealing occurs while hardening steel. The crystallization seeds appear with higher probability in the zones with lower energy. The probability also depends on the steal temperature, which means that when the temperature decreases the probability for a crystal seed to appear increases.

In the synthesis procedure, the unit cost of heat provided to the users has been assumed as the driving force for the simplification of the structure. The probability for a zone to be eliminated (i.e. disconnected from the structure examined at a specific iteration) is high in reason of the difference between the unit cost of heat provided to that zone and the average unit cost. This probability is high if the unit cost of heat for that user is higher than the average value and it is low if the unit cost is lower than the average value.

The unit cost of heat provided to each zone c_j is first calculated and a probability p_j is assigned:

$$p_j = exp\left(\frac{c_j - c}{K \cdot T}\right) \quad (16)$$

This is the probability for the zone j to be deactivated. The numerator in the exponential stands for the difference between the unit cost for the j-th zone and the average cost of heat *c*, K is a constant value (the Boltzmann's constant in the annealing) and T is the number of users connected to the network.

Next step constists in the elimination of a zone. Index j is firstly set to 1, as the zones are processed in order of identification; p is set equal to the probability p_j. A random number x is then extracted. If x is higher than p_j the zone j is deactivated; otherwise p is increased by the value p_{j+1} and j is set to j+1; the test starts again until a value of p grater than x is found.

When a zone is deactivated, a new configuration is found, its heat average cost is calculated as well as the unit cost of all the users still connected. T is diminished of 1 and the procedure starts again. The calculation continues untill all the zones are deactivated.

Each iteration consists of four steps: 1) calculation of the heat average cost of the network and of the unit cost of all the connected users; 2) assignment of a probability cost-based function to all the zones; 3) extraction of a random number x; 4) identification of the zone to be deactivated.

Although it is not a real SA method, deteriorations are allowed as well as improvements: their acceptability is due to probabilities assigned to the zones, i.e. to their unit cost. Due to the probability function, the algorithm can provide each time is run different results. For this reason more than one run is necessary to find a good configuration.

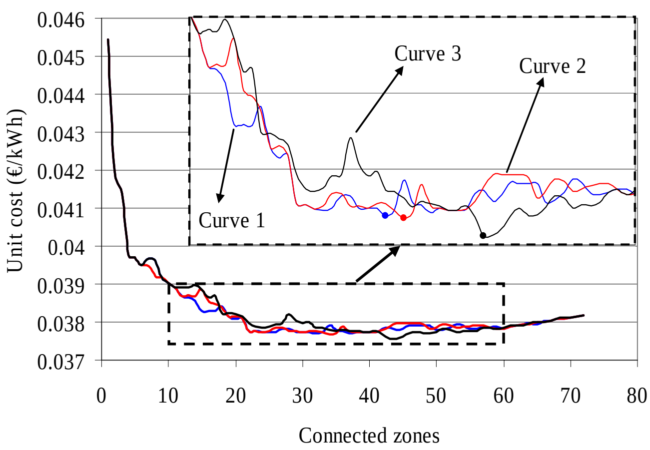

Figure 3. Evolutions of the objective function during the two procedures.

Figure 3 shows how the objective function proceeds while the synthesis procedure is applied. Three different curves are shown. The average unit cost of heat calculated for the initial superstructure is 0.0382 €/kWh. Curve 1 is obtained by disconnecting, each iteration, the zone characterized by the highest unit cost. Using this method, the structure corresponding with the minimum average unit cost is constituted by 31 zones and the average unit cost was 0.0377 €/kWh. Curves 2 and 3 are obtained through application of the proposed approach. In particular, curve 2 corresponds to a general run, which conduces to a non-optimal structure while curve 3 corresponds to a run conducing to the optimal structure. In this configuration there are 43 zones connected and the average unit cost of heat is 0.0375 €/kWh (minimum is highlighted in the zoomed area). This result has been obtained more than once by running the procedure 1000 times.

The optimal configuration obtained with this method, is presented in figure 4.

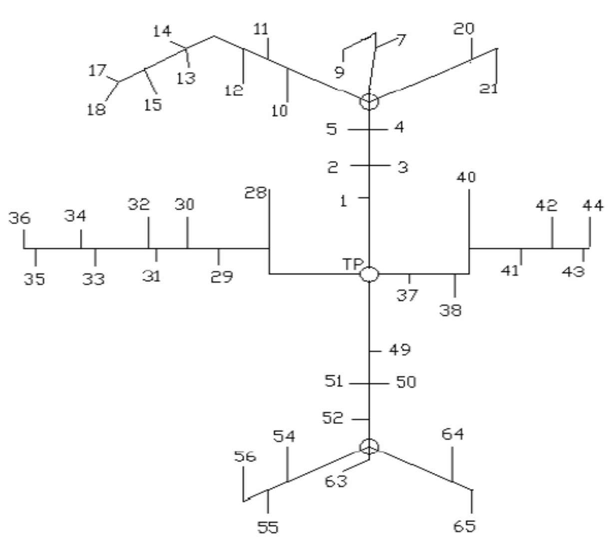

Figure 4. Structure of the optimal network.

CONCLUSION

District heating is an efficient way to provide heat to buildings in a urban area since it can be coupled with cogeneration plants, heat recuperation from waste incineration processes or from industries. High efficiency boilers are generally required as backup systems or for covering pick load. This is a more rational way to provide heat to the users with respect to domestic boilers. In addition, large plants allow a better control of pollutant emissions.

An important problem to be solved when planning a district heating system in a municipality is the extension of the network, which does not necessarily involve the whole town. The area to be connected with the district heating system can be selected according with economic objective functions, such as the minimum cost for the community. For the zones not connected with the network, heat is provided through distributed systems, such as domestic boilers or micro-CHP systems.

The determination of optimal configuration, formulated as a synthesis problem, allows one to decide which areas could be effectively and conveniently fed trough the DHN and which ones through distributed systems. Such design criterion should be always formulated as a first step of the decisional project, since, once the main network is designed, further developments are technically difficult or even impossible if not planned.

In this paper, a procedure for the synthesis of distributing network systems is proposed. The procedure is based on the definition of an initial superstructure, connecting all the possible users. The structure is then simplified through disconnection of some users. A thermoeconomic criterion is considered for the determination of users to be disconnected..

The procedure is similar to Simulated Annealing. It makes use of probability distributions, so that the users characterized by high unit cost have low probability of being included in the determined optimal structure and vice versa.

The application to a portion of the Turin urban area is then proposed. The network is ideal since the selected area is only partially served by district heating. Nevertheless, real data have been used for the applications. The results obtained seam reliable since the optimal configuration has been obtained more than once, having performed 1000 run.

NOMENCLATURE

a_i general polynomial coefficient;
A incidence matrix;
c average unit cost of heat [€/kWh];
C_C annual capital cost [€/year];
c_F unit cost of the heat provided to the network [€/kWh];
cj unit cost of heat provided to the j-th zone [€/kWh];
C_{OP} annual operating cost [€/year];
c_P unit cost of electricity [€/kWh];
C_{net} annual investment cost of the network [€/year];
C_{tot} total annual cost of the network [€/year];
d discount rate;
D_{int} the internal diameter of pipes [mm];
DTD summation of daily temperature difference [°C·day]:
G water mass flow rate [kg/s];
hh number of heating hours per day;
i inflation rate;
K constant term in probability function [€/kWh];
l life of the network [years];
L pipe length [m];
L_p annual electricity consumption [kWh];
p probability;
PC purchase cost [€];
P_0 operational cost at first year [€]
Q_u annual thermal load [kWh];
Q_r annual thermal request [kWh];
Q_z annual thermal load of a zone [kWh];
r volumetric specific losses per unit temperature difference [W/m3K];
T parameter at denominator in the probability function;
v_{max} design velocity in pipe [m/s];
V_z total volume of buildings in the zone [m^3];
X general size variable;
Z vector with cost rate of components [€/s];
α exponent in the cost function;
Δp total pressure losses [Pa];
Φ thermal flow [kW];
η_p average pump efficiency;
Π cost rate [€/s]
$\mathbf{\Pi}$ cost vector [€/s];
ρ density [kg/m^3];
Ψ exergy flow [kW].

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